AAE 412 – Project Using Fluent

Final Report

Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Team 16

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Abstract

In this two dimensional analysis, three cases using given boundary conditions and geometry were observed. The geometry was a simplified version of a converging/diverging nozzle for a supersonic business jet application. The first case was at supersonic cruise conditions, the second case used the same geometry, but at takeoff conditions. The third case added an ejector to the original geometry and was tested at takeoff conditions similar to those in the second case. The goal of the third case was to increase the thrust and mass flow rate as compared to the second case. The major drawbacks of the second case are that there was a large shock in the nozzle and the boundary layer separated after the shock. The third case increased mass flow rate by 8.43%, average velocity at exit by 1.19%, and thrust by 8.58%. These gains were accomplished by inducing two shocks before and after the ejector which led to weaker shocks, lessening the flow and pressure losses from the stronger shock found in case two. The second shock also keeps the boundary layer attached farther down the diverging wall, leading to fewer losses in energy from the stagnated flow after separation. Overall, the ejector has proven to be effective as it increased the thrust by a fairly significant amount.
# Table of Contents

List of Figures .......................................................................................................................... 3
List of Tables .............................................................................................................................. Error! Bookmark not defined.
Introduction ................................................................................................................................. 5
Numerical Solution ...................................................................................................................... 8
Results .......................................................................................................................................... 11
  Case 1 – High Pressure Ratio ................................................................................................. 11
  Case 2 – Low Pressure Ratio ................................................................................................. 17
  Case 3 – Low Pressure Ratio with Ejector ............................................................................ 23
Conclusion .................................................................................................................................. 33
References .................................................................................................................................. 34
Appendix ..................................................................................................................................... 35
  Progress Report 1 ..................................................................................................................... 35
  Progress Report 2 ..................................................................................................................... 37
MATLAB BOUNDARY LAYER CODE ..................................................................................... 58
List of Figures

Figure 1: Test Geometry Sketch
Figure 2: Nozzle Geometry Sketch
Figure 3: Nozzle Geometry
Figure 4: Nozzle Geometry with Ejector Sketch
Figure 5: Ejector Geometry
Figure 6: Case 1 Fine Mesh @ Nozzle Exit
Figure 7: Case 1 Fine Mesh @ Nozzle Throat
Figure 8: Case 1 Fine Mesh Showing Core, Bypass, & Nozzle Wall
Figure 9: Case 1 Fine Grid History of the Residuals
Figure 10: Case 1 Fine Grid Mass Flow Rate @ Nozzle Exit
Figure 11: Case 1 Fine Grid Average X Component of Velocity @ Nozzle Exit
Figure 12: Case 1 Fine Grid Average Pressure @ Nozzle Exit
Figure 13: Case 1 Fine Grid Contour Plot of Static Pressure Inside Nozzle
Figure 14: Case 1 Fine Grid Contour Plot of Static Temperature Inside Nozzle
Figure 15: Case 1 Fine Grid Contour Plot of Mach Number Inside Nozzle
Figure 16: Case 1 Viscous Boundary Layer
Figure 17: Case 1 Fine Grid Wall Cell Distance Coefficient
Figure 18: Case 1 X-Wall Shear Stress
Figure 19: Case 1 Temperature Profile @ Nozzle Exit
Figure 20: Case 2 Fine Grid History of the Residuals
Figure 21: Case 2 Fine Grid Mass Flow Rate @ Nozzle Exit
Figure 22: Case 2 Fine Grid Average X Component of Velocity @ Nozzle Exit
Figure 23: Case 2 Fine Grid Contour Plot of Static Pressure Inside Nozzle
Figure 24: Case 2 Fine Grid Contour Plot of Static Temperature Inside Nozzle
Figure 25: Case 2 Fine Grid Contour Plot of Mach Number Inside Nozzle
Figure 26: Case 2 Fine Grid Cell Distance Coefficient
Figure 27: Case 2 Fine Grid Velocity Vectors at Boundary Layer Separation
Figure 28: Case 2 Fine Grid X-direction Shear Stress Values
Figure 29: Ejector Geometry
Figure 30: Ejector Mesh close-up
Figure 31: Case 3 Fine Grid History of the Residuals
Figure 32: Case 3 Fine Grid Mass Flow Rate @ Nozzle Exit
Figure 33: Case 3 Fine Grid Average X Component of Velocity @ Nozzle Exit
Figure 34: Case 3 Fine Grid Average Pressure @ Nozzle Exit
Figure 35: Case 3 Fine Grid Contour Plot of Static Pressure Inside Nozzle
Figure 36: Case 3 Fine Grid Contour Plot of Static Temperature Inside Nozzle
Figure 37: Case 3 Fine Grid Contour Plot of Mach Number Inside Nozzle
Figure 38: Case 3 Viscous Boundary Layer Separation and Reverse Flow
Figure 39: Case 3 Fine Grid Contour Plot of Static Pressure in Jet Plume
Figure 40: Case 3 Fine Grid Contour Plot of Static Temperature in Jet Plume
Figure 41: Case 3 Fine Grid Contour Plot of Mach Number in Jet Plume
Figure 42: Case 3 Fine Grid Wall Cell Distance Coefficient
List of Tables

Table 1: Case 1 Meshing Parameters
Table 2: Case 1 Boundary Conditions
Table 3: Case 2 Meshing Parameters
Table 4: Case 2 Boundary Conditions
Table 5: Calculated Results at Nozzle Exit for Case 1
Table 6: Calculated Results at Nozzle Exit for Case 2
Table 7: Case 3 Updated Boundary Layers
Introduction

A currently popular topic among civil jet aircraft companies is a supersonic business jet. Not only would such a plane transport VIPs far distances in short amounts of time, but the first company to provide such a design to the customer could see significant profits as a result. A key aspect of any supersonic aircraft is the propulsion system, as it must not only reach and sustain supersonic velocities, but also operate at relatively quiet conditions.

Here, an analysis is provided on a supersonic converging-diverging nozzle for high and low pressure cases. A scale model is used to save computing time by using a fewer amount of grid points. A limitation to this analysis is that a maximum of 500,000 grid points may be used. Figure 1 below outlines a sketch which displays the test case geometry. As this analysis will implement the symmetry plane, the entire domain is 47.5 in x 15 in.

![Domain Size](image)

Figure 1: Test Geometry Sketch

The nozzle geometry itself may differ, as the sketch in Figure 2 on the next page does not provide bound constraints. However, it provides a simple understanding of how the nozzle is set up. Note that the top half is not used in the analysis, as the symmetry plane takes this into account. Figure 3 on the next page shows how the geometry inside the nozzle is distributed. Not including the symmetry plane, there are a total of 5 walls coming from the freestream surrounding, the expanding wall, the core, and 2 from the bypass.
Upon completing the analysis for the high and low pressure cases using the above geometry, an ejector is analyzed. The purpose of an ejector is to provide freestream flow into the diverging part of the nozzle. This will induce a shock earlier in the nozzle, causing a greater mass flow rate with fewer pressure losses. In turn, the thrust exiting the nozzle will be higher. However, the
ejector can only be used at lower velocities, such as during takeoff and climb. At higher velocities, a shock may be induced in the ejector, disrupting flow and potentially damaging the ejector.

A sketch of how the ejector is installed can be seen in Figure 4 below. It is notable how the ejector is fairly tangential to the flow and then curves off into the nozzle flow. Therefore, the actual geometry consists of a similar design, as seen in Figure 5. It consists of a 1 inch set of straight lines which then follow an S-pattern through two arcs, and leads into the nozzle through another set of straight lines. This pattern can be visualized in both Figures 4 and 5. The ejector itself is implemented such that the flow comes in at 1/3 distance downstream of the throat with respect to the diverging part of the nozzle.
Numerical Solution

Before refining our course grid into a fine mesh, we needed to calculate the boundary layer thickness on each of the five walls to ensure that we capture the viscous boundary layer along the walls. The boundary layer thickness (i.e. $\delta$) and corresponding $y^+$ values calculated are shown in Table 1 below. To remain conservative, we used the thickest value computed and we then applied these conditions to each wall individually. As shown below in Figures 6 – 8, one can see that the grid spacing is more clustered near the various walls of the nozzle. A total of 109,941 cells were used in the fine mesh of case 1.

<table>
<thead>
<tr>
<th>Core_S_Wall</th>
<th>Bypass_N_Wall</th>
<th>Bypass_S_Wall</th>
<th>Expanding_Wall</th>
<th>Nozzle_Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 0.1650$</td>
<td>$\delta = 0.2016$</td>
<td>$\delta = 0.2539$</td>
<td>$\delta = 0.15135$</td>
<td>$\delta = 0.3027$</td>
</tr>
<tr>
<td>$y^+_1 = 1.341 \times 10^{-4}$</td>
<td>$y^+_1 = 3.308 \times 10^{-4}$</td>
<td>$y^+_1 = 3.305 \times 10^{-4}$</td>
<td>$y^+_1 = 9.05 \times 10^{-5}$</td>
<td>$y^+_1 = 1.062 \times 10^{-4}$</td>
</tr>
<tr>
<td>$y^+_50 = 6.705 \times 10^{-2}$</td>
<td>$y^+_50 = 1.654 \times 10^{-2}$</td>
<td>$y^+_50 = 1.6525 \times 10^{-2}$</td>
<td>$y^+_50 = 4.525 \times 10^{-2}$</td>
<td>$y^+_50 = 5.31 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 1: Case 1 Meshing Parameters

Figure 6: Case 1 Fine Mesh @ Nozzle Exit
Calculated in Progress Report 2 shown in the appendix, Table 2 shows the boundary conditions applied to the nozzle for Case 1.

<table>
<thead>
<tr>
<th>Bypass</th>
<th>Core</th>
<th>Freestream</th>
<th>Outlet</th>
<th>S_wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_G = 44,535 \text{ Pa})</td>
<td>(P_G = 44,535 \text{ Pa})</td>
<td>(P_G = -82,625 \text{ Pa})</td>
<td>(P_G = -82,625 \text{ Pa})</td>
<td>(P_G = -82,625 \text{ Pa})</td>
</tr>
<tr>
<td>(P_i = 25,620 \text{ Pa})</td>
<td>(P_i = 25,620 \text{ Pa})</td>
<td>(M = 1.5)</td>
<td>(T = 216.65 \text{ K})</td>
<td>(M = 1.5)</td>
</tr>
<tr>
<td>(T = 259.98 \text{ K})</td>
<td>(T = 736.61 \text{ K})</td>
<td>(T = 216.65 \text{ K})</td>
<td>(T = 216.65 \text{ K})</td>
<td>(T = 216.65 \text{ K})</td>
</tr>
</tbody>
</table>

**Table 2: Case 1 Boundary Conditions**
Case 2 was computed in FLUENT using different boundary conditions than Case 1. In order to account for these differences (specifically the variance in Reynolds Numbers along the walls), we needed to calculate the boundary layer thickness on each of the five walls to ensure that we capture the viscous boundary layer. The boundary layer thickness (i.e. δ) and corresponding $y^+$ values calculated are shown in Table 3 below. To remain conservative, we used the thickest value computed and we then applied these conditions to each wall individually.

<table>
<thead>
<tr>
<th>Core_S_Wall</th>
<th>Bypass_N_Wall</th>
<th>Bypass_S_Wall</th>
<th>Expanding_Wall</th>
<th>Nozzle_Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ = 0.1234</td>
<td>δ = 0.1423</td>
<td>δ = 0.1689</td>
<td>δ = 0.1198</td>
<td>δ = 0.3245</td>
</tr>
<tr>
<td>$y^+$ = 3.645e-5</td>
<td>$y^+$ = 6.9e-5</td>
<td>$y^+$ = 5.43e-5</td>
<td>$y^+$ = 3.16e-5</td>
<td>$y^+$ = 1.45e-4</td>
</tr>
<tr>
<td>$y^{50}$ = 1.82e-3</td>
<td>$y^{50}$ = 3.45e-3</td>
<td>$y^{50}$ = 2.715e-3</td>
<td>$y^{50}$ = 1.58e-3</td>
<td>$y^{50}$ = 7.25e-3</td>
</tr>
</tbody>
</table>

### Table 3: Case 2 Meshing Parameters

Calculated in Progress Report 2 shown in the appendix, Table 4 shows the boundary conditions applied to the nozzle for Case 2.

<table>
<thead>
<tr>
<th>Bypass</th>
<th>Core</th>
<th>Freestream</th>
<th>Outlet</th>
<th>S_wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_G$ = 70,928 Pa</td>
<td>$P_G$ = 70,928 Pa</td>
<td>$P_G$ = 0 Pa</td>
<td>$P_G$ = 0 Pa</td>
<td>$P_G$ = 0 Pa</td>
</tr>
<tr>
<td>$P_l$ = 48,591 Pa</td>
<td>$P_l$ = 48,591 Pa</td>
<td>$M = 0.3$</td>
<td>$T = 216.65$ K</td>
<td>$M = 0.3$</td>
</tr>
<tr>
<td>$T = 363.78$ K</td>
<td>$T = 879.135$ K</td>
<td>$T = 303.15$ K</td>
<td>$T = 303.15$ K</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4: Case 2 Boundary Conditions
Results

Case 1 – High Pressure Ratio

The mesh used for Case 1 (described above) utilized the symmetry of the geometry to run FLUENT more efficiently in our calculation. 119,941 quadrilateral cells were used with a minimum orthogonal quality of 0.6626 and maximum aspect ratio of 40.15. FLUENT used a density-based solver and Menter’s SST k-omega viscous model to solve the calculation. The solution methods used Implicit Formulation and Roe-FDS flux type, least squares cell based gradient, and Second Order Upwind scheme. The courant number was set to 5. The solution was initialized from the core boundary condition and converged after 4,840 iterations to an accuracy of 1e-05 as shown in Figure 9 shown below.

Figure 9: Case 1 Fine Grid History of the Residuals
After approximately 4,500, the mass flow rate at the nozzle’s exit leveled off to a value of 4.3794 kg/s. Since only half of the geometry was computed using the symmetry plane, that value needed to be doubled to account for the total mass flow rate calculated at the nozzle’s exit. This value was then used to calculate the thrust produced by the nozzle.

The average x-component of velocity at the nozzle’s exit leveled off after ~3,500 iterations shown in Figure 11 above and the final value computed was 569.7 m/s.
The average pressure at the nozzle’s exit leveled off after ~4,250 iterations shown in Figure 12 above and the final valued computed was 30,062 Pa.

Illustrated in Figure 13, the static pressure at the nozzle exit matches the ambient pressure. This would assume that the flow is perfectly expanded.
Figure 14: Case 1 Fine Grid Contour Plot of Static Temperature Inside Nozzle

The Static Temperature contour plot shown in Figure 14 illustrates the mixing of hot fluid from the core and bypass. The length of the spike between the core and bypass slightly deters the fluids from mixing before the throat; however, mixing is most prevalent in the diverging portion of the nozzle.

Figure 15: Case 1 Fine Grid Contour Plot of Mach Number Inside Nozzle
The contour plot of Mach Number inside the nozzle, shown in Figure 15 above, illustrates that the flow is choked at the throat ($\sim M = 1$). Mach Number increases through the diverging portion of the nozzle to $\sim M = 2$ at the nozzle exit. Figure 15 also illustrates that there is no boundary layer separation and that the flow is perfectly expanded. There are no shocks in the nozzle.

**Figure 16: Case 1 Viscous Boundary Layer**

Figure 16 shows the x-velocities vectors and viscous boundary layer along the diverging portion of the nozzle. There is no boundary layer separation along any of the nozzle walls for Case 1.
Figure 17 shown on the previous page show the $y^+$ values along the walls of the nozzle. The interior walls all have $y^+$ values of ~50 concluding that there is no separation in the boundary layer.

Figure 18: Case 1 X-Wall Shear Stress
Figure 18 shown above illustrate the x-wall shear stress along each of the walls of the nozzle. All wall shear stress values are greater than 0 further reiterations that there is no boundary layer separation.

![Static Temperature vs. Curve Length](image)

**Figure 19: Case 1 Temperature Profile @ Nozzle Exit**

Figure 19 shown on the pervious page illustrates the Temperature Profile at the nozzle exit. This is only half of the total nozzle length since symmetry was used to run FLUENT more efficiently. The temperature is highest near the center of the nozzle, dramatically decreases moving towards the nozzle wall and then gradually increases near the nozzle wall. The temperature distribution is continuous and relatively uniform. Table 6 below shows the exit condition values and calculated thrust.

<table>
<thead>
<tr>
<th>Exit Condition</th>
<th>Case 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Flow Rate</td>
<td>8.7592 kg/s</td>
</tr>
<tr>
<td>Average Velocity</td>
<td>569.7 m/s</td>
</tr>
<tr>
<td>Average Pressure</td>
<td>30,062 Pa</td>
</tr>
<tr>
<td><strong>Thrust</strong></td>
<td><strong>5067.67 N</strong></td>
</tr>
</tbody>
</table>

**Table 5: Calculated Results at Nozzle Exit for Case 1**

\[
\text{Thrust} = \dot{m}v_e + P_eA_e
\]

**Case 2 – Low Pressure Ratio**

The mesh for Case 2 utilized the symmetry of the geometry and used 139,698 quadrilateral cells. FLUENT was run with a density based solver. The residual values monitored where the energy equation and the SST k-omega viscous model. The solution method used Implicit Formulation,
Roe-FDS flux type, Least Squares cell based Gradient, and a First Order Upwind scheme. The Courant Number was set at 20 and the Under Relaxation Factors where left at their default values. The solution was initialized with the freestream boundary conditions and took 3,175 iterations to converge to an accuracy of 1e-05 for all monitored residuals as shown in the below figure.

![Figure 20: Case 2 Fine Grid History of the Residuals](image1.png)

The mass flow rate through the plane at the nozzle exit leveled out shortly after 1,500 iterations, as shown in figure 18 above. The mass flow value for half of the nozzle was 4.577 kg/s. This value was doubled when used in the thrust analysis to account for the entire nozzle.

![Figure 21: Case 2 Fine Grid Mass Flow Rate @ Nozzle Exit](image2.png)
One of the surface monitors used plotted the mass weighted average velocity. The average x-component velocity of the flow at the nozzle exit, shown in the figure above, leveled off slightly before 1,500 iterations. The final value was 298.97 m/s.

The Static Pressure contour plot in the above figure, illustrates the shock where the lower pressure blue suddenly jumps to the higher pressure light green. The uniformity in the color grid at the nozzle exit indicates that the flow was perfectly expanded at the exit because there was no pressure change between the flow coming out the nozzle and the flow from the free stream.
The length of the mixing spike decreased the amount of mixing that went on before the throat, shown in the above figure. This phenomenon is most prevalent in the above figure of the static temperature inside the nozzle.

The Mach Number contour plot, shown in the above figure, does a good job illustrating the supersonic shock in the nozzle and the separation of the boundary layer after the shock. The shock occurs where the high mach number, red on the color grid, jumps to a lower mach number, green. The dark blue along the walls after the shock, is the slowest moving fluid in the entire region and is representative of the stagnation that happens after the boundary layer separates.
Figure 26: Case 2 Fine Grid Wall Cell Distance Coefficient

The above figure shows the $y^+$ values along the inner walls of the nozzle. The black dots are from the wall upstream of the throat and the red dots designate the diverging wall after the throat. For Case 2 we did not use a wall function, so it is expected that all $y^+$ values be less than 1.

Figure 27: Case 2 Fine Grid Velocity Vectors at Boundary Layer Separation
The above plot show the velocity vectors along the diverging wall of the nozzle. At the left of the figure, the boundary layer is accurately depicted and as the flow travels downstream, the layers closest to the wall lose enough energy that they begin to travel in the opposite direction, causing the S shape that is seen in the right most column of vectors.

![Figure 28: Case 2 Fine Grid X-direction Shear Stress Values](image)

The above figure shows the wall shear stress in the x-direction along the inner walls of the nozzle. As in previous figures, the black dots are from the converging wall before the throat and the red dots are from the diverging wall. The boundary layer separation happens when the shear stress drops below zero, right around 13 inches in the above figure. Table 6 below shows the exit condition values and calculated thrust.

![Table 6: Calculated Results at Nozzle Exit for Case 2](table)

\[
\text{Thrust} = \dot{m}v_e + P_e A_e
\]
Case 3 – Low Pressure Ratio with Ejector

In order to increase efficiency during takeoff and climb, an ejector is implemented in the nozzle design. This ejector can be added anywhere along the expanding part of the nozzle and should increase mass flow rate, and thus overall thrust. At higher speeds, it is not as efficient because the supersonic freestream velocity could create shocks within the ejector. Therefore, the ejector is closed during high velocities, and analysis will only be conducted in a low pressure environment, as the one described in case 2.

In selecting the ejector geometry, arbitrary round values were selected which seemed to provide good results after 3 trial and error runs. Therefore, the ejector width $\Delta L$ has a value of 1 in, and the ejector enters the nozzle at 33.33% downstream of the throat compared to the entire expanding section. A combination of lines and arcs is used to redistribute the flow as well. This is used to smooth the flow coming into the nozzle. The overall ejector geometry can be seen in Figure 29 below.

![Figure 29: Ejector Geometry](image)

The boundary layers along the walls must be recalculated as the velocity and length, and thus the Reynolds number have changed. Table 7 below summarizes the boundary layers that have changed, which include the new ejector geometry boundary layers as well. Once again, a $y^+$ value of 50 was selected.

<table>
<thead>
<tr>
<th>Wall</th>
<th>$\delta$ (in)</th>
<th>$y_1$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expanding_W_Wall</td>
<td>0.0022</td>
<td>3.8514*10^3</td>
</tr>
<tr>
<td>Expanding_E_Wall</td>
<td>0.00126</td>
<td>3.5935*10^3</td>
</tr>
<tr>
<td>Nozzle_W_Wall</td>
<td>0.00589</td>
<td>1.7761*10^4</td>
</tr>
<tr>
<td>Nozzle_E_Wall</td>
<td>0.0035</td>
<td>1.6549*10^4</td>
</tr>
<tr>
<td>Ejector_W_Wall</td>
<td>0.00311</td>
<td>0.00703</td>
</tr>
<tr>
<td>Ejector_E_Wall</td>
<td>0.00311</td>
<td>0.00703</td>
</tr>
</tbody>
</table>

Table 7: Case 3 Updated Boundary Layers

As before, the boundary layers were applied in an exponentially growing grid, where values are very narrow near the wall and far spaced apart near the center. As the boundary layer looks very similar as in case 2 for all walls, these will not be shown. Instead, the focus is on the boundary
layer distribution at the ejector, which is displayed below in Figure 30. An important aspect is that the freestream flow coming into the ejector follows the mesh, so there are no cusps. Similarly, as the ejector exit meets the nozzle flow, the mesh takes on the flow trajectory once again. The overall mesh included a total of 152,209 cells.

![Ejector Mesh close-up](image)

**Figure 30: Ejector Mesh close-up**

The ejector case was run using a first-order upwind method, similar to that in case 2. The pressure and temperature values at the core and bypass inlet, as well as the freestream conditions are unchanged from those in case 2, as case 3 should provide a comparison between the two results.

Figures 31–34 below and on the following pages show the residual results as the case converged. As seen, the solution converged to an error of only $10^{-3}$ in continuity after ~ 3,200 iterations. The remaining residuals converge to values between $10^{-5}$ and $10^{-4}$, all with a decreasing slope. It can therefore be assumed that the case would eventually reach machine precision if it had run for more iterations.
The mass flow rate, x-velocity, and average pressure at the nozzle exit are displayed in Figures 32–34. They all reach a nearly converged value at around 3,000 iterations already. It is interesting to note the change in order of magnitude near 300 iterations, which explains the spikes created in the residual plot from Figure 31 above.

Figure 31: Case 3 Fine Grid History of the Residuals
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Figure 32: Case 3 Fine Grid Mass Flow Rate @ Nozzle Exit

Figure 33: Case 3 Fine Grid Average X Component of Velocity @ Nozzle Exit
As a result of the converging case, the contours inside the nozzle can be seen in Figures 35–37 on the following pages. They represent the contours of pressure, temperature, and Mach number inside and around the nozzle, respectively. As is evident by the pressure and Mach number contours, there is a shock just upstream of the ejector, much further upstream than for case 2. However, this also means that there is a weaker shock, as the Mach number has not reached as high of a velocity as it did for case 2, where a strong shock was present. Furthermore, a second shock occurs downstream of the ejector, coincidentally at the same Mach number as the initial shock. Once again, the losses are not as significant as those for case 2 due to the weaker shock. Lastly, the temperature contour displays how the hot core temperature mixes with the cold bypass flow, and how the ejector flow once more decreases overall temperature by mixing with the even colder freestream flow.
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 35: Case 3 Fine Grid Contour Plot of Static Pressure Inside Nozzle

Figure 36: Case 3 Fine Grid Contour Plot of Static Temperature Inside Nozzle
Another notable point is flow in the west wall of the ejector. It appears that there is a stagnation point which causes circulation and thus reverse flow. This is evident in Figure 38 below. Not only does this affect the flow coming in from the ejector, but also the flow just upstream thereof, as it is disrupted by this reversed flow coming in. However, the reverse flow only goes so far as to meet with the flow at the downstream end of the first shock, so it does not affect the overall flow as significantly.
The contours outside the nozzle exit and concerning the jet plume are shown in Figures 39–41 on the following pages. Similar to above, they represent the contours of pressure, temperature, and Mach number, respectively. As shown in the pressure contour, the pressure at the exit of the nozzle equalizes to that of the surrounding as it should. When observing the temperature and Mach number in the jet plume, it is clear that underexpanded flow occurs as soon as the flow leaves the nozzle exit.

Figure 39: Case 3 Fine Grid Contour Plot of Static Pressure in Jet Plume
Figure 40: Case 3 Fine Grid Contour Plot of Static Temperature in Jet Plume

Figure 41: Case 3 Fine Grid Contour Plot of Mach Number in Jet Plume
The thrust calculation for this case is shown below, where the values used are the exit conditions of the nozzle.

\[
\text{Thrust} = \dot{m}_e + P_e A_e = \left(9.9966 \, \frac{kg}{s}\right) \left(302.56 \, \frac{m}{s}\right) + \left(32,737 \, \frac{Pa}{2 \, \text{in}^2}\right) = 3,019 \, N
\]

As compared to the 2,760 N thrust from case 2, this constitutes an 8.58% thrust increase. Furthermore, the mass flow rate increased from the original 9.154 kg/s to a similar increase of 8.43%. As both mass flow rate and thrust increased, the ejector has proven to be an effective design addition.

As shown in the boundary layer analysis, a maximum thickness was assumed in order to provide conservative results. For this, the \(y^+\) values were set at a value of 50, as a reasonable range lies between 30 and a few hundreds. However, as the viscous boundary layer forms in a square root curve, the \(y^+\) values change. Therefore, Figure 42 on the next page provides the actual \(y^+\) values along the nozzle walls. It is notable that most values fall within the reasonable range, with the exception of the ejector walls and the expanding wall downstream of the ejector where separation occurs, which is at around 15.5 in.
Conclusion

For this analysis, two dimensional analyses of three cases using given boundary conditions and geometry was performed. The geometry was a simplified version of a converging/diverging nozzle for a supersonic business jet application. All analyses were run with meshes totaling more than 100,000 cells, viscous flow conditions, and Menter’s SST k-omega turbulence model. The first case was at supersonic cruise conditions, Mach 1.5 at 40,000 ft. The second case used the same geometry, but at takeoff conditions, Mach 0.3 at sea level on a warm day, 0 AGL and 303.15 K. The major drawbacks of the second case are that there is a large shock in the nozzle and the boundary layer separates after the shock. A sub goal of the second case was to accurately capture the boundary layer separation. This was accomplished using exponential stretching laws to congregate nodes in the area closest to the wall. The third case added an ejector to the original geometry and was tested at takeoff conditions identical to those in the second case. The ejector added one inch to the length of the nozzle bringing the total length of the nozzle to 18.5 inches.

The goal of the ejector was to increase the thrust and mass flow rate from the second case. Analysis determined that when comparing the base geometry results to those of the ejector results, thrust increased from 9.154 to 9.997 kg/s, an increase of 8.43%, the average velocity at the nozzle exit increased from 298.98 to 302.56 m/s, an increase of 1.19%, and thrust increased from 2,760 to 3,019 N, an increase of 8.58%. These gains were accomplished by inducing two shocks at a low Mach number before and after the ejector which led to weaker shocks, lessening the mass flow and pressure losses as compared to those of the stronger shock found in case two. The second shock also keeps the boundary layer attached farther downstream of the diverging wall, leading to fewer losses in energy from the stagnant flow after boundary layer separation. Overall, the ejector has proven to be effective as it increased the thrust by a fairly significant percentage.
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

References


Appendix

Progress Report 1
AAE 412 Progress Report 1
John Bartos
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Team 16

The interior wall contours were created from the schematics given in Problem Set 8. Our team chose to use the optional symmetry plane from the problem statement to simplify the geometry. We only used half of the inflow from the engine and only the bottom inflow from the bypass. We arbitrarily chose the mixing spike between the engine and bypass flows to be 7.5 inches long. We used 11 grid points to create the geometry.
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application
Progress Report 2

Aeronautics and Astronautics
Purdue University

Date: November 15, 2013
To: Prof. Gregory Blaisdell
From: Stefan Linforder (Team Captain)
John Bartos
Ryan Harmeyer
Subject: AAE 412 Progress Report 2 (Task 2)

Introduction

This progress report represents the completion of task 2 for our project using Fluent. As you recall, we are using Fluent to analyze flow from a converging-diverging nozzle for a supersonic business jet application at high and low pressure ratios.

Completed Research

For task one, we created a coarse grid for the nozzle (without the ejector) using ICEM CFD. For task two, we ran Fluent with the coarse grid for inviscid flow through the nozzle (without the ejector). Our results Cases (i)—(iii) using a coarse grid are as follows:

Coarse Grid – Inviscid Flow
Case (i) – High Pressure Ratio

Assumptions
- Flow subsonic upstream of the throat
- Flow is close to perfectly expanded
- Isentropic flow throughout the nozzle ($\gamma = 1.4$)
- Pressure at the nozzle exit equal to the pressure in the atmosphere

Bypass Stream (2)
- $P_0/P_{inf} = 7.8$ (stagnation pressure)
- $T_0/T_{inf} = 1.2$ (stagnation temperature)

Core Stream (1)
- $P_0/P_{inf} = 7.8$ (stagnation pressure)
- $T_0/T_{inf} = 3.4$ (stagnation temperature)

Freestream (pressure far field)
- Standard atmosphere @ 40,000 ft
- Mach = 1.5 (Mach number)

Freestream:

From Anderson’s book *Fundamentals of Aerodynamics*

$P_{inf} = 18,700 \text{ Pa} \rightarrow -82,625 \text{ Pa} @ 40,000 \text{ ft}$
\( T_{\inf} = 216.65 \text{ K} @ 40,000 \text{ ft} \)

**Bypass:**

\[
\frac{P_0}{P_\infty} = 7.8
\]

\( P_{0,e} = 145,860 \text{ Pa} \rightarrow P_{0,g,e} = 44,535 \text{ Pa} \)

\[
A = h \times 1 \text{ in}
\]

\( A^* = 1.186 \text{ in}^2 \)

\( A_i = 1.719 \text{ in}^2 \)

\[
\frac{A_i}{A^*} = \sqrt{\frac{1}{M_i^2\left(\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2}M_i^2\right)\right)^{\gamma+1}}} = 1.45
\]

\( M_i = 0.45 \)

\[
\frac{P_0}{P_i} = \left(1 + \frac{\gamma - 1}{2}M_i^2\right)^{\frac{\gamma}{\gamma - 1}} = 1.149
\]

\( P_i = 126945 \text{ Pa} \rightarrow P_{g,I} = 25620 \text{ Pa} \)

\[
\frac{T_0}{T_\infty} = 1.2
\]

\( T_0 = 259.98 \text{ K} \)

**Core:**

\[
\frac{P_0}{P_\infty} = 7.8
\]

\( P_0 = 145860 \text{ Pa} \rightarrow P_{0,g,e} = 44535 \text{ Pa} \)

\( P_i = 126945 \text{ Pa} \rightarrow P_{g,I} = 25620 \text{ Pa} \)

\[
\frac{T_0}{T_\infty} = 3.4
\]

\( T_0 = 736.61 \text{ K} \)

**Coarse Grid Inviscid Flow High Pressure Ratio Fluent Results:**
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 1: Coarse Grid History of the Residuals

Figure 2: Coarse Grid Mass Flow Rate through nozzle (w/o ejector)
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 3: Coarse Grid Average X Component of Velocity through nozzle (w/o ejector)

Figure 4: Coarse Grid Average Pressure through nozzle (w/o ejector)
Figures 2 – 4 show the history of the mass flow rate, the mass weighted average of the x component of velocity, and the average pressure respectively using our coarse grid for Case (i)—High Pressure Ratio without ejector. Within Fluent, we used the SIMPLE algorithm as our numerical method to compute our results. After ~550 iterations shown in Figure 1 history of residuals, the following average values were computed:

Mass Flow Rate = 10.461 [kg/s]
X Component of Velocity = 585.63 [m/s]
Pressure = 43,878 [Pa]

From these quantities, we were able to compute the thrust generated by the nozzle.

\[
Thrusted = \dot{m} v_e + P_e A_e = \left(10.461 \frac{kg}{s}\right) \left(585.63 \frac{m}{s}\right) + (43,878 \text{ Pa})(2 \text{ in}^2) = 6,182.89 \text{ N}
\]

Below are our contour plots of pressure, temperature, and Mach number inside the nozzle as well as separate plots of those quantities in the jet plume downstream of the nozzle for Case (i)—High Pressure Ratio without ejector.
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 6: Coarse Grid Contour Plot of Temperature Inside Nozzle (w/o ejector)

Figure 7: Coarse Grid Contour Plot of Mach Number Inside Nozzle (w/o ejector)
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

**Figure 8:** Coarse Grid Contour Plot of Pressure in the Jet Plume Downstream of the Nozzle (w/o ejector)

**Figure 9:** Coarse Grid Contour Plot of Temperature in the Jet Plume Downstream of the Nozzle (w/o ejector)
Figure 10: Coarse Grid Contour Plot of Mach Number in the Jet Plume Downstream of the Nozzle (w/o ejector)
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Case (ii) and (iii) – Low Pressure Ratio

Assumptions
- Shock in the nozzle
- Flow is choked
- Subsonic isentropic flow upstream of the throat \( (\gamma = 1.4) \)

Bypass Stream (2)
- \( P_0/P_{\text{inf}} = 1.7 \) (stagnation pressure)
- \( T_0/T_{\text{inf}} = 1.2 \) (stagnation temperature)

Core Stream (1)
- \( P_0/P_{\text{inf}} = 1.7 \) (stagnation pressure)
- \( T_0/T_{\text{inf}} = 2.9 \) (stagnation temperature)

Freestream (pressure far field)
- Standard atmosphere @ sea level + 15K (temperature)
- Mach = 0.3 (Mach number)

Freestream:

From Anderson’s book *Fundamentals of Aerodynamics*

\( P_{\text{inf}} = 101,325 \text{ Pa} \rightarrow 0 \text{ Pa} @ 0 \text{ ft} \)
\( T_{\text{inf}} = 288.15 \text{ K} @ 0 \text{ ft} + 15\text{K} \rightarrow 303.15 \text{ K} \)

**Bypass:**

\[
\frac{P_0}{P_{\infty}} = 1.7
\]

\( P_{0,e} = 172,253 \text{ Pa} \rightarrow P_{0,g,e} = 70,928 \text{ Pa} \)

\[
A = h \times 1 \text{ in}
\]

\( A^* = 1.186 \text{ in}^2 \)
\( A_i = 1.719 \text{ in}^2 \)

\[
\frac{A_i}{A^*} = \sqrt{\frac{1}{M_i^2} \left( \frac{2}{\gamma + 1} (1 + \frac{\gamma - 1}{2} M_i^2) \right)^{\frac{\gamma + 1}{\gamma - 1}}} = 1.45
\]

\( M_i = 0.45 \)

\[
\frac{P_0}{P_i} = \left( 1 + \frac{\gamma - 1}{2} M_i^2 \right)^{\frac{\gamma}{\gamma - 1}} = 1.149
\]

\( P_i = 149,916 \text{ Pa} \rightarrow P_{g,i} = 48,591 \text{ Pa} \)
\[
\frac{T_0}{T_\infty} = 1.2
\]

\(T_0 = 363.78 \text{ K}\)

**Core:**

\[
\frac{P_0}{P_\infty} = 1.7
\]

\(P_0 = 172,253 \text{ Pa} \rightarrow P_{0,g,e} = 70,928 \text{ Pa}\)

\(P_i = 149,916 \text{ Pa} \rightarrow P_{g,I} = 48,591 \text{ Pa}\)

\[
\frac{T_0}{T_\infty} = 2.9
\]

\(T_0 = 879.135 \text{ K}\)
Coarse Grid Inviscid Flow Low Pressure Ratio Fluent Results:

![Graph showing residuals history](image1)

Figure 11: Coarse Grid History of the Residuals

![Graph showing mass flow rate](image2)

Figure 12: Coarse Grid Mass Flow Rate through nozzle (w/o ejector)
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 13: Coarse Grid Average X Component of Velocity through nozzle (w/o ejector)

Figure 14: Coarse Grid Average Pressure through nozzle (w/o ejector)
Figures 12 – 14 show the history of the mass flow rate, the mass weighted average of the x component of velocity, and the average pressure respectively using our coarse grid for Case (ii) and (iii)—Low Pressure Ratio without ejector. Within Fluent, we used the SIMPLE algorithm as our numerical method to compute our results. After ~600 iterations shown in Figure 11 history of residuals, the following average values were computed:

Mass Flow Rate = 10.862 [kg/s]
X Component of Velocity = 278.1 [m/s]
Pressure = 34,122 [Pa]

From these quantities, we were able to compute the thrust generated by the nozzle.

\[ Thrust = m \dot{v}_e + P_e A_e = \left( 10.862 \frac{kg}{s} \right) \left( 278.1 \frac{m}{s} \right) + (34,122 \text{ Pa})(2 \text{ in}^2) = 3,064.75 \text{ N} \]

Below are our contour plots of pressure, temperature, and Mach number inside the nozzle as well as separate plots of those quantities in the jet plume downstream of the nozzle for Case (ii) and (iii)—Low Pressure Ratio without ejector.
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 16: Coarse Grid Contour Plot of Temperature Inside Nozzle (w/o ejector)

Figure 17: Coarse Grid Contour Plot of Mach Number Inside Nozzle (w/o ejector)
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Figure 18: Coarse Grid Contour Plot of Pressure in the Jet Plume Downstream of the Nozzle (w/o ejector)

Figure 19: Coarse Grid Contour Plot of Temperature in the Jet Plume Downstream of the Nozzle (w/o ejector)
Creation of Fine Grid
Local Skin Friction Coefficient

From Whites’ book *Viscous Fluid Flow*

\[ \delta = \frac{0.37 x}{Re_x^{0.2}} \]

\[ C_f = \frac{0.058}{Re_x^{0.2}} \]

where Re\(_x\) is the Reynolds number based on distance from the start of the boundary layer. Re\(_x\) is defined as Re\(_x\) = \(x \cdot v_{\text{ref}} / \nu\), where \(v_{\text{ref}}\) is the velocity outside the boundary layer and \(\nu = \mu / \rho\) is the kinematic viscosity (\(\mu\) is the dynamic viscosity, \(\rho\) is the density).

The skin friction coefficient is used to find the value of \(y^+\) for the first grid point off the wall, since \(y^+\) is defined as \(y^+ = y \cdot u_\tau / \nu\), and \(u_\tau = \sqrt{\tau_w / \rho}\), where \(\tau_w\) is the wall shear stress, which is related to the skin friction coefficient through \(c_f = \tau_w / ((1/2) \rho u_{\text{ref}}^2)\). For a fine grid we need to have \(y^+ < 1\), while if wall functions are used we should have \(y^+ > 30\) (or 50) and < 300.

**Case (i) –High Pressure Ratio**

Assuming rectangular boundary layer shape, maximum thickness is at the end of each wall, so \(x = L\).
From Online Tables,
\( \rho_{\text{inf}} = 0.3 \text{ kg/m}^3 \) @ 40,000 ft
\( \mu_{\text{inf}} = 1.4\times10^{-5} \text{ Pa}\cdot\text{s} \) @ 40,000 ft

**Core_S_Wall:**
L = 7.5 in
From Contour plots,
\( v = 330 \text{ m/s} \)

\[
Re_c = \frac{\rho vL}{\mu} = 1,347,100
\]

\[
\delta = \frac{0.37L}{\sqrt{Re_c}}
\]

\( \delta = 0.165 \text{ in} \)

\[
C_f = \frac{0.058}{\sqrt{Re_c}} = 0.0034
\]

\[
C_f = \frac{\frac{\tau}{2\rho v^2}}{}
\]

\( \tau = 56.3205 \text{ Pa} \)

\[
v_T = \frac{\sqrt{\frac{\tau}{\rho}}}{13.7016 m/s}
\]

\( y = \frac{y^+ \mu}{\rho v_T} \)

\( y = 1.341\times10^{-4} \text{ in} \)

**Bypass_N_Wall:**
L = 7.5003 in
From Contour plots,
\( v = 121 \text{ m/s} \)

\( \delta = 0.2016 \text{ in} \)

\( y = 3.308\times10^{-4} \text{ in} \)

**Bypass_S_Wall:**
L = 10.0057 in
From Contour plots,
\( v = 121 \text{ m/s} \)

\( \delta = 0.2539 \text{ in} \)

\( y = 3.308\times10^{-4} \text{ in} \)

**Expanding_Wall:**
L = 7.511 in
From Contour plots,
\( v = 511 \text{ m/s} \)

\( \delta = 0.15135 \text{ in} \)
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

\[ y = 9.05 \times 10^{-5} \text{ in} \]

**Nozzle Wall:**
- \( L = 17.5 \text{ in} \)
- From Contour plots,
- \( v = 470 \text{ m/s} \)

\[ \delta = 0.3027 \text{ in} \]
\[ y = 1.062 \times 10^{-4} \text{ in} \]

**Case (ii) and (iii) – Low Pressure Ratio**

- From Online Tables,
  - \( \rho_{\text{inf}} = 1.225 \text{ kg/m}^3 \)
  - \( \mu_{\text{inf}} = 1.789 \times 10^{-5} \text{ Pa*s} \)

**Core_S_Wall:**
- \( L = 7.5 \text{ in} \)
- From Contour plots,
- \( v = 440 \text{ m/s} \)

\[
Re_c = \frac{\rho v L}{\mu} = 5,739,500
\]
\[
\delta = \frac{0.37L}{\sqrt{Re_c}}
\]

\[ \delta = 0.1234 \text{ in} \]

\[
C_f = \frac{0.058}{\sqrt{Re_c}} = 0.0026
\]

\[
C_f = \frac{\tau}{\frac{1}{2} \rho v^2}
\]

\[ \tau = 305.9601 \text{ Pa} \]

\[
v_t = \sqrt{\frac{\tau}{\rho}} = 15.8039 \frac{m}{s}
\]

\[ y = 3.64 \times 10^{-5} \text{ in} \]

**Bypass_N_Wall:**
- \( L = 7.5003 \text{ in} \)
- From Contour plots,
- \( v = 216 \text{ m/s} \)

\[ \delta = 0.1423 \text{ in} \]
\[ y = 6.9 \times 10^{-5} \text{ in} \]

**Bypass_S_Wall:**
- \( L = 10.0057 \text{ in} \)
- From Contour plots,
\[ v = 291 \text{ m/s} \]
\[ \delta = 0.1689 \text{ in} \]
\[ y = 5.43 \times 10^{-3} \text{ in} \]

**Expanding Wall:**

L = 7.511 in

From Contour plots,

\[ v = 515 \text{ m/s} \]

\[ \delta = 0.1198 \text{ in} \]

\[ y = 3.16 \times 10^{-5} \text{ in} \]

**Nozzle Wall:**

L = 17.5 in

From Contour plots,

\[ v = 104 \text{ m/s} \]

\[ \delta = 0.3245 \text{ in} \]

\[ y = 1.45 \times 10^{-4} \text{ in} \]

Figures 21 and 22, shown below, illustrate the fine grid created with points clustered near the solid walls to capture the viscous boundary layers.
Figure 21: Fine Grid in the Interior of the Nozzle for Core_S_Wall, Bypass_N_Wall, Bypass_S_Wall, Nozzle_Wall

Figure 22: Fine Grid in the Interior of the Nozzle for Bypass_S_Wall, Nozzle_Wall
Remaining Research

Based upon our solutions found on the coarse grid for Cases (i)—(iii), we now need to run Fluent on the fine grid (without the ejector) to find solutions for both high and low pressure ratio conditions specified previously. We will use Menter’s SST $\kappa-\omega$ turbulence model in our calculations. After we have a converged solution, we will evaluate the boundary layer velocity profiles at several locations to determine if our solution is well resolved.

Once we completed analyzing the baseline geometry, we will then change the design to make an ejector.

Conclusion

This progress report concludes the completion of task 2 for our project using Fluent. We successfully found a converged solution in Fluent with a coarse grid for inviscid flow through a nozzle (without the ejector). We are currently on schedule and should be ready for our oral presentation 12/3-6/2013.

References


MATLAB BOUNDARY LAYER CODE

% AAE 412 Team 16
% Finding Boundary Layer Thickness for each wall

clc
clear

%% High Pressure Ratio
% case (i)
% Core_S_Wall, Bypass_N_Wall, Bypass_S_Wall, Expanding_Wall, Nozzle_Wall

c = [7.5 7.5 10.0057 7.511 17.5]; % total wall length in in
c = 0.0254*c; % total wall length in m
v = [330 121 121 511 470]; % approximate wall velocity in m/s

% case i) 40,000 ft
rho_40k = 0.3; % density in kg/m^3
mu_40k = 1.4e-5; % dynamic viscosity in N*s/m^2
Re_40k_c = rho_40k*v.*c/mu_40k; % Reynold's number for maximum thickness
C_f_40k = 0.058./Re_40k_c.^0.2; % Coefficient of Friction
delta_40k = 0.37*c./Re_40k_c.^0.2; % Boundary Layer thickness in m
delta_40k = delta_40k/0.0254; % Boundary Layer thickness in in

tau_40k = 1/2*rho_40k*v.^2.*C_f_40k;

v_tau_40k = sqrt(tau_40k/rho_40k);

% assume y+ = 1
y_plus_40k = 1;
y_40k = mu_40k./(rho_40k*v_tau_40k*y_plus_40k);

%% Low Pressure Ratio
% case (ii) & (iii) @ 0 ft
v = [440 216 291 515 104]; % approximate wall velocity in m/s

rho_0 = 1.225; % density in kg/m^3
mu_0 = 1.789e-5; % dynamic viscosity in N*s/m^2

Re_0_c = rho_0*v.*c/mu_0;
C_f_0 = 0.058./Re_0_c.^0.2;
delta_0 = 0.37*c./Re_0_c.^0.2; % Boundary Layer thickness in m
delta_0 = delta_0/0.0254; % Boundary Layer thickness in in

tau_0 = 1/2*rho_0*v.^2.*C_f_0;
v_tau_0 = sqrt(tau_0/rho_0);

% assume y+ = 1
y_plus_0 = 1;
y_0 = mu_0./(rho_0*v_tau_0*y_plus_0);

y_0 = y_0/0.0254; % first grid cell distance in in
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Case 1: Boundary Layer Thickness along Wall for Bypass_N_Wall

Case 2: Boundary Layer Thickness along Wall for Bypass_S_Wall

Case 3: Boundary Layer Thickness along Wall for Core_S_Wall

Case 4: Boundary Layer Thickness along Wall for Expanding_Wall

Case 5: Boundary Layer Thickness along Wall for Nozzle_Wall

Team 16
Supersonic Business Jet (SSBJ) Converging-Diverging Nozzle Application

Case i) Boundary Layer Thickness along Wall for Bypass_S_Wall

Case ii) Boundary Layer Thickness along Wall for Core_S_Wall

Case ii) Boundary Layer Thickness along Wall for Expanding_Wall

Case ii) Boundary Layer Thickness along Wall for Nozzle_Wall

Team 16